Available online at www.sciencedirect.com

SCIENCE DIRECT®

Historia Mathematica 33 (2006) 43–62

HISTORIA MATHEMATICA

www.elsevier.com/locate/yhmat

Reading Luca Pacioli's *Summa* in Catalonia: An early 16th-century Catalan manuscript on algebra and arithmetic ☆

Javier Docampo Rey

Departament de matemàtiques, IES Berenguer d'Anoia, Ctra. De Palma-Alcúdia s/n, 07300 Inca, Balearic Islands, Spain

Available online 15 June 2005

Abstract

This paper focuses on an anonymous Catalan manuscript of the early 16th century dealing with algebra and commercial arithmetic. More than half of it consists of a series of notes concerning parts of Luca Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Proportionalità*, while most of the rest is related to Joan Ventallol's commercial arithmetic (1521). The use of a new kind of diagrams to work with equations is especially remarkable. The article throws new light on the cultivation of algebra in the Iberian peninsula before 1552, when the treatise on algebra by Marco Aurel, *Libro Primero de Arithmética Algebrática*, was first printed in Spanish.

Resum

Aquest article se centra en l'àlgebra i l'aritmètica comercial d'un manuscrit català anònim de començaments del segle setze. Més de la meitat del manuscrit consisteix en una sèrie de notes dedicades a algunes parts de la *Summa de Arithmetica, Geometria, Proportioni et Proportionalità* de Luca Pacioli, mentre que la resta està relacionada en la seva major part amb l'aritmètica mercantil de Joan Ventallol (1521). L'ús d'un nou tipus de diagrames per treballar amb equacions és especialment important. L'article posa nova llum en el cultiu de l'àlgebra a la Península Ibèrica abans del 1552, quan va ser imprès el primer tractat d'àlgebra en castellà, el *Libro Primero de Arithmética Algebrática* de Marco Aurel.

MSC: 01A40; 97-03

Keywords: Commercial arithmetic; Algebra; Catalonia; Renaissance; Luca Pacioli; Joan Ventallol

☆ Revised version of a paper presented at the International Workshop *The Origins of Algebra: From al-Khwarizmi to Descartes*, Barcelona, March 27–29, 2003.

E-mail address: javdoc@hotmail.com.

0315-0860/$ – see front matter
doi:10.1016/j.hm.2005.03.004

1. The context

It is hardly surprising that the most important influences in Catalan treatises on arithmetic of the late medieval and early Renaissance periods came from the French–Provençal area and from Italy.[1] The geographical and cultural proximity allowed connections between the practical arithmetic cultivated in the Catalan and Provençal areas, as can be seen in the case of Francesc Santcliment's *Summa de l'art d'Aritmètica* (Barcelona, 1482). The Italian mathematical knowledge would have been transmitted by the many Italian merchants who were established in Catalonia, Valencia, and Majorca and also by Catalan merchants who made long journeys in the Mediterranean and even lived for some time in Italian cities.[2]

Almost all the arithmetic books printed in Europe before 1500 were texts on commercial arithmetic.[3] This indicates that there was a wide audience for this genre, which made this kind of works economically attractive for printers. Most of these texts were published in Italy or in Germany. As is well known, the first printed text dealing with algebra is Luca Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Proportionalità* (Venice, 1494).[4] Its 600 pages contain most of the mathematical knowledge of the medieval period from both the commercial arithmetic and the "speculative" traditions.[5] It also includes the first printed treatise on bookkeeping.

Pacioli's *Summa* was highly influential both inside and outside of Italy. According to F.K.C. Rankin, an anonymous *Trattato de aritmetica* of the 16th-century contains some chapters that are explicitly attributed to the *Summa*, while Niccolò Simi Bolognese, a mathematics teacher of the University of Bologna, wrote the *Annotazioni sopra l'aritmetica di Fra Luca da Borgo S. Sepolcro* in 1544. Both manuscripts contain corrections and explicative notes on Pacioli's most difficult parts, suggesting that their authors used them in their teaching.[6] The *Summa* was also an important reference work for many other 16th-century algebraists such as Pedro Nunes,[7] Tartaglia, Cardano, Bombelli, or the Sienese teacher Dionigi Gori, and its influence was also felt in Germany and England.[8] Since Pacioli's *Summa* is a mathematical encyclopedia rather than a teaching text, it is not the best book from a pedagogical point of view. It is therefore hardly surprising that its contents were taught through manuscripts such as Niccolò Simi's or the Catalan text to be discussed in this article.

---

[1] Most of the texts on commercial arithmetic and algebra of the period 1300–1600 are preserved in Italy, where more than 400 from the period have been found (see [Van Egmond, 1980]). For European commercial arithmetic in the late medieval and early Renaissance periods, see [Benoit, 1989; Davis, 1960; Franci and Toti Rigatelli, 1982; Spiesser, 2003; Swetz, 1987; Van Egmond, 1980].

[2] It is interesting to find an abacus teacher from Pisa, Christoforo Grillo, living in Barcelona in the middle of the 15th century (see [De la Torre y Del Cerro, 1971, 252–253]). The presence of Italians in the Catalan area in the late Middle Ages is well documented (see, for instance, [Ferrer, 1980]).

[3] [Malet and Paradís, 1984, 45].

[4] A second edition was printed in 1523 in Toscolano.

[5] This was not the first time that material on commercial arithmetic appeared beside more "theoretical" chapters, such as the theory of proportions or the classification of irrationals. This also happened, for instance, in the *Trattato di praticha d'arismetrica* of M^{o} Benedetto (1463) (see [Arrighi, 1965]).

[6] The anonymous manuscript is in the Biblioteca Ambrosiana of Milan (MS P. 114 Sup.), and Simi's notes are preserved in the Bologna University Library (MS 595 N (5)) [Rankin, 1992, 110].

[7] This Portuguese author criticizes the *Summa* but recognizes its influence on the authors of his time, including himself: "(...) fue la Summa de Arithmética y Geometría, que compuso Fray Lucas de Burgo, excellente Arithmético, de la qual todos después nos avemos aprovechado" [Nunes, 1567, f. 323v].

[8] [Rankin, 1992, 112].

The second printed commercial arithmetic in Europe, Francesc Santcliment's *Summa de l'art d'Aritmètica*, was written in Catalan and published in 1482 in Barcelona.[9] Santcliment's book has clear French–Provençal connections,[10] and the same is the case for Juan de Ortega's commercial arithmetic of 1512.[11] On the other hand, the influence of Luca Pacioli's *Summa* is especially strong in Juan Andrés' arithmetic (Valencia, 1515)[12] and very important in Gaspar Nicolas' *Tratado da prática D'arismétyca* (Lisboa, 1519)[13] and Joan Ventallol's *Pràtica mercantívol* (Lyon, 1521).[14] The manuscript to be presented here is another example of Pacioli's influence in the Iberian peninsula in the first half of the 16th century and shows that Pacioli's algebra was also influential there.

Only four Catalan manuscripts on commercial arithmetic from before 1500 have been found so far.[15] The earliest one is a fragment of a text of ca. 1390 that still uses the Roman numeration system and is related to calculation with the abacus.[16] Two others are preserved in Italy and can be dated to ca. 1440–1450: one of these contains only a few solved problems, and the other is a long list of unsolved exercises systematically ordered, most of which deal with money exchange and the calculation of prices.[17] Only the fourth manuscript consists of a complete treatise: the *Llibre que esplica lo que ha de ser un bon mercader* (ca. 1490)[18] is a merchants' handbook that includes a commercial arithmetic. The text of this arithmetic has important coincidences with Francesc Santcliment's *Summa* and also displays significant Italian influences.[19]

While the authorship of three of the four above-mentioned manuscripts is completely unknown, it is possible that the arithmetical part of the *Llibre que esplica*... was written by an important Barcelonese merchant of the second half of the 15th century named Gaspar Montmany, since he is the author of a good part of the handbook. In any case, evidence shows that this arithmetical part is related to the teaching of a certain Galceran Altimir, who is quoted at the beginning as a master who divides arithmetic into four

9 This is also the first mathematics book printed in the Iberian peninsula. A modern critical edition is available (see [Malet, 1998]).

10 See [Malet, 1998, 33–39, 63–72].

11 The *Compusición de la arte de la arismetica y juntamente de geometría*... was composed in Castilian by the Palentine Dominican friar Juan de Ortega and first published in Lyon in 1512 (other editions are quoted in [López Piñero and Bujosa, 1984, 263–266] and in [Navarro Brotóns et al., 1999, 241–243]). This work is briefly commented upon by Salavert [1990, 69–70] and Malet and Paradís [1984, 123–127].

12 The *Sumario breve de la prática de la arithmética de todo el curso de larte mercantívol bien declarado: el qual se llama maestro de cuento* was written in Castilian by Juan Andrés, a priest from Zaragoza who declares explicitly that he has taken most of his book from Pacioli's *Summa* (f. 58v). Andrés' text is briefly treated by Salavert [1990, 70–72]. A facsimile edition is available (Valencia, 1999).

13 Nicolas's is the first printed Portuguese arithmetic.

14 [Docampo, 2004, 538–539]. Joan Ventallol was a Majorcan and published his commercial arithmetic in Catalan. A Ph.D. dissertation with a detailed analysis of the works on arithmetic by Francesc Santcliment, Juan de Ortega and Joan Ventallol has recently been presented in Toulouse (see [Labarthe, 2004]). I have not yet been able to consult this work.

15 These texts are analyzed in detail in [Docampo, 2004].

16 Ms. 454, Biblioteca de Catalunya, Barcelona.

17 Both manuscripts are preserved together in Ms. 102 (A.III 27), Biblioteca degl'Intronati di Siena. They were first edited and briefly commented upon by Arrighi [1982].

18 Ms. Diversos 37B/2, *Arxiu del Regne de Mallorca*, Palma de Mallorca.

19 The Italian influence can be seen, for instance, in the use of the form *di* for the preposition *de* ("of") on several occasions, and in the way of dealing with multiplication and division, which reminds us of the program of a Pisan abacus school (ca. 1420, see [Arrighi, 1965–1967]). For further details, see [Docampo, 2004, 198–199, 207–215].

parts (addition, subtraction, multiplication, and division).[20] Altimir taught arithmetic in Barcelona at least from 1460 onward, since in that year he appears in a legal contract for the teaching of practical arithmetic and the writing of commercial letters.[21] In this contract he is described as a *scriptore littera rotunda*, a specialist of round script. Thus he was simultaneously a *scriptor* and a teacher of mathematics. Notice that the same connection between teaching mathematics and writing is found elsewhere. Nicolas Chuquet, for one, is described in the fiscal registers of Lyon first as an *escrivain* and later as an *algorist*. It seems that the term *escrivain* was used in that city to designate those teachers who educated the sons of the bourgeoisie.[22] The Catalan term for *scriptor* is *escrivà*, so this seems to indicate that the kind of teaching activity of Altimir and Chuquet was not too different.

While we do not yet have any evidence for the existence of abacus schools[23] in Barcelona, some comments by contemporary observers suggest that centers comparable to the Italian ones were not established in that city.[24] Apart from Altimir, two other teachers of commercial arithmetic in 15th-century Barcelona have been identified so far. One of them is a Pisan abacus teacher named Christoforo Grillo, already mentioned. The other is Francesc Santcliment, who taught arithmetic in Barcelona around 1482, as he declares at the end of his *Summa*.[25] Unfortunately, we do not have any document with information about schools or pupils related to the teaching of Grillo or Santcliment.

None of the Catalan manuscripts mentioned above includes an algebra section or even a single problem solved explicitly by an equation. A Castilian manuscript on commercial arithmetic of around 1400 contains precisely one problem solved by a first-degree equation.[26] However, it was not until 1552 that the first known Spanish algebra treatise, the *Libro Primero de Arithmética Algebrática* by Marco Aurel, was printed in Valencia.[27] Aurel knew Pacioli's *Summa*,[28] but his book is mostly influenced by German algebra texts—not surprisingly, since he was of German origin. At the beginning of his treatise he declares that: "As what I am treating is a new thing, and never seen, nor declared and perhaps neither understood nor printed in Spain, I dare to treat it, and write it in a language so odd to mine."[29] But in fact

20 *Llibre que esplica...*, f. 76v. Altimir is also said to be the person in charge of the books of King Ferran (Ferdinand) II. It is known from other sources that he received this assignment in 1481 (Del Rey, MR, reg. 920, f. 56, *Arxiu de la Corona d'Aragó*, Barcelona).

21 Altimir concludes this contract with a 20-year-old pupil called Pere de Mont-Real, to whom he will teach arithmetic in exchange for his work as a servant (see [Hernando i Delgado, 2002, 426]).

22 See [Benoit, 1988, 97; Flegg, 1988, 60–61].

23 The abacus schools or *bottege d'abaco* provided the future merchants with the necessary skills in commercial arithmetic and have been documented in cities such as Florence, Lucca, Pisa, or Siena. For Florentine abacus schools, see [Arrighi, 1965; Goldthwaite, 1972; Ulivi, 2001]. For abacus schools in Siena, see [Franci and Toti Rigatelli, 1982].

24 See [Docampo, 2004, 132–134]. This question needs to be fully researched.

25 "(...) fou acabada la summa present sobre l'art d'aritmètica per mi, Francesc Santcliment, en la insigne ciutat de Barcelona aquella ensenyant" [Malet, 1998, 352 (f. 135v)].

26 This treatise is preserved in a 16th-century copy and has recently been edited; see [Caunedo and Córdoba, 2000] (the mentioned problem appears on p. 200 of this edition).

27 Aurel deals with basic commercial arithmetic in about a quarter of his book, while the rest is devoted to a systematic exposition of the theory of equations and topics such as square numbers, irrationals and binomials. The book has not yet been systematically studied, although it has been briefly commented upon in [López Piñero, 1979, 176; Malet and Paradís, 1984, 127; Salavert, 1985, 165–167; 1990, 77].

28 Parts of Pacioli's work are quoted by him: see [Aurel, 1552, f. 77v, f. 125v].

29 "Assí que por ser cosa nueva lo que trato, y jamás vista, ni declarada: y podrá ser que ni aún entendida, ni imprimida en España, me he atrevido a tratarla, y escrivirla en lengua tan por entero repugnante a la mía" [Aurel, 1552, f. 3r].

algebra was *not* unknown in the Spanish kingdom before Aurel published his treatise. In his arithmetic of 1515, Juan Andrés announces his plans to compose a treatise on algebra,[30] of which nothing else is known. However, the manuscript described in this article proves definitely that algebra was cultivated in Catalonia already before the publication of Aurel's book.

2. Description of the manuscript

Ms. 71 of the Monastery of Sant Cugat's section in the *Arxiu de la Corona d'Aragó* (Barcelona) is an anonymous Catalan manuscript containing notes on algebra and commercial arithmetic.[31] It can be dated to the first part of the 16th century and consists of 160 sheets of paper of 150 by 210 mm, numbered in modern times. An unknown part of the contents is missing, as some pages have been completely or partially torn up. Some of the sheets are clearly older than others, although all the mathematical contents are written in Catalan by the same hand (with some small parts in Latin). We can also find a short passage about astrology written in Castilian. There is no formal introduction nor any reference to the author's name in the preserved pages. The contents do not follow any specific order, and a few paragraphs are repeated on different pages.

On some pages of the manuscript there are calligraphic exercises in a different kind of script, apparently by someone who copied notarial documents in order to practice his writing on pages left unused when the mathematical notes were written. In an inserted account of activities and expenses there is mention of the years 1532 and 1533. In a calligraphic exercise on still another page, we find a note that makes reference to a tax collector working in Vilafranca in 1520 and 1521.[32] Since in all probability these texts were later additions to the manuscript, the years they contain suggest latest possible dates for the composition of the mathematical notes. Furthermore, the script of the main text is very similar to scripts in dated writings of the period 1498–1528 which are also preserved in the *Arxiu de la Corona d'Aragó*.[33] Finally, the watermarks in some pages are also contained in contemporary documents that are explicitly dated. We may therefore assume that the manuscript was written between 1500 and 1530, and most probably in the second decade of the 16th century.[34]

The manuscript contains a wide range of material on arithmetic and algebra, but it is not a formal treatise. It consists of a series of notes written on separate sheets of papers or in small notebooks that

---

30 [Andrés, 1515, f. 51v].

31 The manuscript was formerly preserved in the archive of the Monastery of Sant Cugat. It is briefly described in [Cifuentes, 2002, 306; Miquel, 1937, 118–119]. From here on we will mostly refer to Ms. 71 simply as "the manuscript."

32 "Capbreulevador de les Rendes dels perueres de la present vila de Villafranqua qui comença lo primer del mes de abril del any 1520 e finirà lo darrer del mes de marҫ del any 1521, he vense per temps de un any" [Ms. 71, f. 82v]. We believe the placename to refer to Vilafranca de Conflent, in today's Southern France.

33 These are contained in *Cajas para la historia del Archivo* ("Boxes for the History of the Archive"), n^{o} 4.

34 The help of Jaume Riera of the *Arxiu de la Corona d'Aragó* has been essential in dating the manuscript. Some clear coincidences in problems on square numbers and money exchange show a connection between the manuscript and Juan Andrés' arithmetic (see [Docampo, 2004, 364–369, 479–482]). If the Aragonese priest knew the Catalan manuscript when he composed his treatise, it is obvious that these parts of the manuscript would have been written before 1515. However, it is also possible that the author of the manuscript translated some of these problems from Andrés' treatise after that date.

were put together later on, perhaps by some descendant or disciple of the author. They could be part of the notes taken by a teacher to prepare his lectures,[35] or the results of the study of an advanced student. A number of illustrative diagrams and several comments to explain certain procedures point toward the first of these possibilities. For instance, after an explanation of the resolution of a problem by means of algebra, the author writes: “And I have wanted to do all this discourse to show how to argue in algebra. But you will solve it in a shorter way by false position, as you will see in the following figure.”[36]

Examination of the manuscript shows that more than half of it is devoted to the study of several topics from Luca Pacioli’s *Summa*, most of the time paraphrasing its contents. The remainder is a collection of materials on commercial arithmetic of unknown origin, although most of it is clearly related to Ventallol’s *Pràtica mercantívol*.

It is evident that the author does not limit himself to translating Pacioli’s work. He clearly understands what he is writing. Occasionally, he gives alternative ways to solve problems. For instance, there is a problem about buying and selling which is solved by Pacioli by a first-degree equation.[37] Although the algebraic procedure is included in the manuscript, a solution without algebra is also presented. The author does not use a method of false position but considers the linear relation between the two magnitudes involved (length and price). This technique is applied by Pacioli in other examples, such as number 43 on f. 173r of the *Summa*.

Several Latin quotes, titles, and even complete paragraphs from the *Summa* appear in the manuscript, where they are adapted or summarized. This attests to the author’s knowledge of Latin.

Some of the commercial problems in the manuscript were adapted from Pacioli’s book to the Catalan context. For instance, in one of the exchange problems the author uses Barcelonese *sous* instead of Pacioli’s *tornesi* and Perpinyanese *sous* instead of *ravegnani*.[38] This might suggest that the author intended to compose a treatise for local merchants by adding adaptations of parts of the *Summa* to material on commercial arithmetic that he had available. A table with equivalences of Barcelonese money with monetary units from Valencia, Naples, Majorca, Palermo, Rome, etc. is included (f. 146r) and denominations from Barcelona are often used in the problems. This suggests that the manuscript is directed to an audience from this city.

It is possible that the author of the manuscript taught commercial arithmetic to future merchants and also gave lectures on the most “theoretical” part of the *Summa* to some highly-cultivated urban aristocrats with intellectual interests. However, on this question we can do nothing but speculate.

A detailed study of Ms. 71 of Sant Cugat is one of the chapters of my Ph.D. dissertation, which was presented at the University of Santiago de Compostela in September 2004.[39] In the present article I summarize the most important results of that chapter.

[35] If this were the case, we would surely be talking about private lectures, since the contents of the manuscript were not, as far as we know, part of any “official” curriculum in early 16th-century Catalonia.

[36] “E tot aquest discurs he volgut fer per mostrar lo modo de arguir en àlgebra. Emperò més breu la faràs per una falsa posició com veuràs per la figura següent” [Ms. 71, f. 71v]. The mentioned figure is not included in the manuscript.

[37] Ms. 71, f. 74v. It appears in [Pacioli, 1494, f. 196v].

[38] Ms. 71, ff. 77v–78r; [Pacioli, 1494, f. 171r–v].

[39] See [Docampo, 2004, 307–563].

3. Authorship: the connection with Joan Ventallol

The Majorcan Joan Ventallol published a commercial arithmetic in Catalan in 1521 titled *Pràtica mercantívol*.[40] Very little is known about this author apart from what can be found in his treatise: that he was from *Ciutat de Mallorques* (today's Palma de Mallorca) and that he paid the expenses of the publication of his book. There is a notarial document of 1508 in which an *escrivà* called Joan Ventayol (sic) is mentioned as a witness in Majorca.[41] To be sure, we cannot say that this person was the author of the *Pràtica mercantívol* without any further information. However, it is certainly possible, since, as we have seen, terms such as *escrivà* and *scriptor* were associated with Nicolas Chuquet and Galceran Altimir, whose professional activity was probably very similar to that of Joan Ventallol.

The *Pràtica mercantívol* does not include algebra. Yet Ventallol's familiarity with Luca Pacioli's book is evident in other places, since he turns to the *Summa* for the "speculative" part of his own work: theory of numbers, theory of proportions, and results on square numbers. The influence of the *Summa* can also be seen in the chapters on progressions and roots, and in the introduction of the book.

Ventallol quotes Juan de Ortega and Juan Andrés several times to criticize certain aspects of their works.[42] On the other hand, he was also influenced by French–Occitan mathematics: some of the recreational problems he includes are very close to those found in the collection of problems that follows Chuquet's *Triparty*.[43] Ventallol also includes Chuquet's "rule of the intermediate terms" (*rigle des nombres moyens*).[44] Moreover, one of the chapters of the *Pràtica mercantívol* is called *De oposicions e ramocions*,[45] a terminology very typical of French and Occitan treatises,[46] and also to be found in Santcliment's *Summa de l'art d'Aritmètica*.[47]

Returning to the manuscript Ms. 71 of Sant Cugat, it can be noted that almost all the problems on commercial arithmetic that do not come from Pacioli's *Summa* are to be found in Ventallol's treatise, often with the same numerical values and a very similar style of explanation and vocabulary, even if the illustrative diagrams used in the problems of partnership are different. In both texts the word *manco* for "minus" is used and the question of most of the problems starts with the formula *jo us deman* ("I ask

40 The text of this arithmetic is edited in [Mesquida, 1996], together with a philological study of the book. There also exists a facsimile edition, published in Palma in 1985.

41 [Mesquida, 1996, 19–20].

42 Ventallol also mentions Hyerònimo d'Angest, the author of a Latin treatise about proportions, in the context of his definition of *proportion* [Ventallol, 1521, f. 113r].

43 See [Marre, 1881, 456–458]. These problems can be found in [Ventallol, 1521, ff. 119v–120v]. They have the same titles as Chuquet's and the explanations are given in a very similar way. Nicolas' Chuquet's *Triparty en la science des nombres* [Ms. fds franç. 1346, Bibl. Nat. in Paris] was composed in Lyon in 1484. See, for instance, [Itard, 1979–1980; Malet and Paradís, 1984, 71–88; Flegg, 1988; Beaujoan, 1988].

44 [Ventallol, 1521, ff. 118v–119r]. He calls it the "Retgla dita de mediació entre lo més e lo manco," and the text is very close to that of Chuquet (see [Marre, 1880, 653–654]). Estienne de la Roche also includes the rule in his *Larismethique nouellemant composee...*, published in Lyon in 1520, and calls it "regle de mediation entre le plus et le moins" (see [Marre, 1880, 576]). It would be interesting to compare the texts concerning these topics in the *Pràtica mercantívol*, published one year later in the same city, and Estienne de la Roche's arithmetic. However, it is outside the scope of this article to investigate this aspect in more detail.

45 [Ventallol, 1521, ff. 110r–111r]. The chapter consists of a list of six problems concerning someone's will.

46 See [Beaujoan, 1988, 80; Benoit, 1982, 220; Sesiano, 1984, 61–63; Van Egmond, 1988, 136–137].

47 See [Malet, 1998, 36, 71–72, 328–329 (ff. 123v–124r)].

you"), instead of the more simple form *deman* of other contemporary Catalan texts. Both texts also generally use the word *rays* for "root" instead of *rael*.[48]

We can see an example of the strong resemblance of the manuscript and Ventallol's treatise by comparing the beginning of the section devoted to partnership in both works.[49]

In Ms. 71 of Sant Cugat:

> Aprés de haver dade fi e conclusió en la retgla de tres de la suficiència del que toca a la pràtica mercantívoll, en aquest capítol tractaré de la diversitat de les Compenyie[s] ab (...), que les demés se poden fer per la retgla de 3. Emperò en algunes és manaster tenir asment als pactes e avinensas que entra ells fan perquè és necessari tenir l'orda de lur concòrdia com veuràs per los següents eximplis.
>
> Diu la retgla sens pactes: Moltiplica lo guany per la mesa de cascú e lo que vindrà perteix per totes les meses ensemps.

In the *Pràtica mercantívol*:

> Aprés de haver tractat de la retgla de .3., per dar compliment a l'art negociativa, en aquest capítol tractaré de la diversitat de les compenyies a bé que les demés se pugan fer per la retgla de .3. Emperò en algunes és manaster tenir esment als pactes e avinences que entra ells fan. (...), car lavores es manaster haver respecte a la mesa y al temps. E si pactes hi a entre ells, lavores és manast[e]r antendre la forma e manera dels pactes, axí com en los aximplis següents poràs largament veura.
>
> E diu la retgla general sens pactes: Moltiplica lo guany per la mesa de quiscú e lo que·n vindrà perteix per tots ensemps.

A similar way of expressing the rule can be found in Santcliment's *Summa* and in some French and Occitan treatises,[50] especially in the *Kadran aux marchans*, a French commercial arithmetic composed (at least partially) in Bilbao in 1485.[51]

Some of the material that the author of the manuscript takes from Pacioli's *Summa*, especially that related to the theory of proportions and square numbers, also appears in Ventallol's treatise. In most of these parts the text of the manuscript is closer to the *Summa* than to the *Pràtica mercantívol*, so we may surmise that the author of the manuscript was not using Ventallol's treatise, and it seems likely that the manuscript was used by Ventallol. On the other hand, there is an example in the section devoted to roots (dealing with the calculation of the square root of $10\frac{72}{300}$) that does not appear in the *Summa*, but can be found in the *Pràtica mercantívol* explained in the same manner.[52]

48 *Raíz* in modern Castilian and *arrel* in modern Catalan. J.A. Mesquida considers *rays* as coming from Castilian [Mesquida, 1996, 342]. However, we also find this word in the *Compendy de la praticque des nombres*, a manuscript composed in the South of France in 1471 (see [Spiesser, 2000, 366]) and in Francés Pellos' *Compendion de l'abaco* (printed in 1492), while Ortega and Andrés use *raíz* and *rayz*, respectively. So the use of *rays* could be related to a French–Occitan connection.

49 Ms. 71, f. 111r; [Ventallol, 1521, f. 61r]. In the transcriptions we have added punctuation marks, accents, hyphens, and diaereses. We have also separated words, dissolved abbreviations, corrected spelling mistakes (when they make the text difficult to understand), and normalized u/v and capital letters. Omitted words and letters are inserted in brackets. We have always written *ells* instead of *els* (when the word stands for the pronoun "they") and changed n into m when it appears before b or p.

50 See [Malet, 1998, 70; Spiesser, 2003, 39].

51 "On doit multiplier chune mise pour le gaing et partix ce que en viendra pour toutes les mises ensemble" [Certain, 1485, f. 41v].

52 Ms. 71, ff. 28v–29r; [Ventallol, 1521, f. 105r, *retgla 9*].

With all the above evidence in mind we can tentatively attribute the notes in the manuscript to Joan Ventallol. He would then have written some parts of the *Pràtica mercantívol* using notes contained in the manuscript, and probably considered algebra as too advanced a topic for most of the readers of his printed book. Perhaps he planned, like Juan Andrés, to write a separate algebra treatise based on Pacioli's text.

4. Contents of the manuscript

Roughly speaking, the first two-thirds (ff. 1r–119v) of Ms. 71 of Sant Cugat are almost completely devoted to Pacioli's *Summa*,[53] while the rest (ff. 120r–159v) is unrelated to it. Most of the problems are completely solved, and in many cases the solving process is described in a very detailed way.

The topics taken from the *Summa* are square numbers, theory of proportions, operations with binomials, classification of irrational quantities, operations with nonexact roots, and theory of equations and its applications. More than once the author of the manuscript takes only some solved examples from the *Summa* and not the general explanation that precedes them.

Below, after listing the main notations used in the manuscript, we will focus on three features that allow us to characterize Ms. 71 and its author. First of all, we examine the way in which the algebra from the *Summa* is treated in the manuscript. Secondly, we give an overview of the part of the manuscript that is not taken from the *Summa*. Finally, we will discuss a problem on proportions that is found in the *Summa* as well as with Ventallol and provides one of the strongest indications for the latter's authorship of the manuscript. A type of diagram of coefficients which is used to work with equations is the most original feature of Ms. 71 and will therefore be treated separately in Section 5.

4.1. Notations

The notations used in the manuscript are mostly those from Pacioli's book, which are the usual ones in Italian algebra. For instance, "2 .co. més 3 .ce. més 12 n°" means $2x + 3x^2 + 12$. The main notations used are listed in Table 1.[54] Note that the abbreviations for "plus" and "minus" are different from the $\tilde{p}$ and $\tilde{m}$ used in the *Summa*.

4.2. The treatment of algebra

The author of the manuscript introduces the main elements of the theory of equations: number (*nombre*), unknown (*cosa*), and square of the unknown (*cens*). The six basic types of second-degree equations are given in a non-normalized form, as in Pacioli's *Summa*:

$$ax^2 = bx \qquad ax^2 = b \qquad ax = b$$
$$ax^2 + bx = c \qquad bx + c = ax^2 \qquad ax^2 + c = bx.$$

53 However, both the diagrams used for the multiplication of binomials and the way of introducing the basic algebraic elements suggest that the author knew at least one other Italian algebra text besides the *Summa* (see [Docampo, 2004, 431–432, 442–443]).

54 The word "nombre" used in Table 1 also refers to units in the independent term of an algebraic expression.

Table 1
Main notations used in Ms. 71 of Sant Cugat, *Arxiu de la Corona d'Aragó* (Barcelona)

Concept	Description	Notation
Nombre	Number/unit	n^{o}
Cosa	Unknown	.co.
Cens	Square of the unknown	.ce.
Cubo	Cube of the unknown	.cu.
Quantitat	Quantity/second unknown	$\tilde{q}_3^{at}$
Més	Plus	m^{e}
Manco	Minus	m^{a}
Rays/rael	Root in a general sense	℞
Rays quadra	Square root	℞
Rays cúbica	Cubic root	℞ cu.[55]
Rays quarta	Quartic root	℞℞
Rays universal	Root affecting more than one term	℞ ụ

The author explains how to reduce other equations to one of the six main types. He does not include geometrical constructions for the resolution of the last three canonical equations, while Pacioli does.[56] Pacioli, like Fibonacci, accepts the two positive roots in the sixth case when $(\frac{b}{2})^2 > c$ (considering a normalized equation). While in the manuscript this is not stated in the description of the resolution of this kind of equation, nor in the example that follows it, the two positive solutions are accepted in problems. This is also the case in another example for the sixth rule found in another section of the manuscript.[57]

Other algebraic topics included are equations with algebraic fractions or roots, the introduction of a second unknown, the introduction of powers of the unknown up to the 29th degree, and long lists of products of such powers. We also find the resolution of some types of fourth-degree equations.[58] The equations $ax^4 + bx^2 = cx$ (*cens de cens e cens egual a cosa*) and $ax^4 + bx = cx^2$ (*cens de cens e cosa egual a cens*) are labeled "impossible," as in Pacioli's *Summa*. However, the author declares later (again following Pacioli) that equations like these could not be solved *yet*, as in the case of the quadrature of the circle.[59] As has already been noticed, Pacioli did not claim that the cubic could not be solved, but that a rule for it had not yet been found.[60]

The author includes a good number of applications of algebra to the main types of problems of commercial arithmetic: partnership, barter, exchange, computation of interest, etc. However, most of these problems are algebraic exercises in a "commercial disguise" rather than actual applications of algebra to the resolution of commercial problems.

---

55 Pacioli also uses ℞.q. [Pacioli, 1494, f. 118r].

56 These constructions were already absent from most Italian algebra texts of the 14th century [Franci and Toti Rigatelli, 1988, 29].

57 Ms. 71, ff. 94v–97v.

58 Ms. 71, f. 94v.

59 "Emperò quant en lo teu agolament tu trobaras termens de diversos entrevals entra ells desproporcionats, diràs que l'art encare a tal cosa no ha dat modo, com de la quadratura circuli" (Ms. 71, f. 99r, translated from [Pacioli, 1494, f. 150r]).

60 This fact has already been observed in [Franci and Toti Rigatelli, 1985, 64–65; Van Egmond, 1978, 159]. It has been asserted that when Pacioli used the word "impossible" in this case, he only meant that it was not possible to solve these types of equations along the lines of the quadratic [Rankin, 1992, 14].

4.3. Commercial arithmetic: material not related to the Summa

On ff. 108r–113v of Ms. 71 there are some problems on money exchange and partnership that do not appear in the *Summa*, and from f. 122r onward we find more material on commercial arithmetic that is not taken from Pacioli's book either. These two parts of the manuscript show French and Provençal as well as Italian influences and, as has already been mentioned, contain much material that also appears in Ventallol's *Pràtica mercantívol*. The basic arithmetical operations (except for addition) are treated and the methods for checking them explained. The algorithm of division used in the manuscript is the same as that found in the more or less contemporary Catalan arithmetics such as Santcliment's. It is basically identical with the "galley" method, with minor differences in the placement of the numbers.[61]

Problems on exchange, calculation of interest, and partnership are also included. A section titled *Extraordinàries* (ff. 149r–153v) is particularly noteworthy: it consists of a short collection of problems that are not the usual commercial calculations; some of them concern mixtures of grains and are solved by the usual methods in the problems of alligation. There is also a rule for "guessing a number" that someone has thought of.[62]

4.4. A relevant problem on proportions

The author of the manuscript takes several properties of proportion from the *Summa*. Seven of them are called *meravelles* (marvels), as in Pacioli's book. These rules also appear in Ventallol's *Pràtica mercantívol*, in a very similar style. However, a detailed comparison of the three texts makes clear that the author of the manuscript was reading and following Pacioli's treatise, not the *Pràtica mercantívol*.[63]

We will now have a look at a relevant problem appearing in the *Summa*, in the Ms. 71, as well as in the *Pràtica mercantívol*[64]: three quantities must be found in geometric proportion such that their sum is equal to the sum of the quotients of the divisions of 36 by each of the three. I have included, as an Appendix A, the common part of the text of the example as described in the three versions. The transcription will again show the strong resemblance between the texts and will remind us of the rhetoric style of that time.

In modern notation, the problem asks the reader to find three quantities a, b, and c in geometrical progression such that

$$\frac{36}{a}+\frac{36}{b}+\frac{36}{c}=a+b+c.$$

[61] The "galley" method was an algorithm of Indian origin. It was the most popular method of division in Europe before the 17th century; see [Rankin, 1992, 159–168]. This method, in the form used by Santcliment, is explained in [Malet, 1998, 357–358; Pla, 1998, 234–238].

[62] This same rule appears in the *Liber abaci* (see [Sigler, 2002, 427–428]). Similar rules are to be found in many other medieval and Renaissance texts.

[63] See [Docampo, 2004, 380–381, 664–666].

[64] [Pacioli, 1494, ff. 88v–89r]; Ms. 71, ff. 17r–18r; [Ventallol, 1521, ff. 117v–118r].

This problem is included in our three sources as an application of the following property (expressed in modern terms): given a quantity q and three quantities a, b, and c in geometrical progression such that $\frac{q}{a}+\frac{q}{b}+\frac{q}{c}=a+b+c$, then $q=b^2(*)$.[65]

The resolution in the manuscript consists of the following steps:

1. "The second quantity has to be the square root of 36 [by $(*)$], so it is 6."

2. "The remainder up to 36 is 30, so we have to divide 30 in 2 parts such that their product is 36. These will be x (*1.co.*) and $30-x$ (*30 manco 1.co.*)."

3. Thus $x\cdot(30-x)=36$, i.e., $30x=x^2+36$ (*30.co. eguals a .1.ce. més 36*).

4. "Follow the chapter and you will find $x=15-\sqrt{189}$ and this will be the first number. The third one will be $15+\sqrt{189}$."

Here, "follow the chapter" means "do the necessary operations to solve the second-degree equation," which leads to $x=\frac{30}{2}-\sqrt{(\frac{30}{2})^2-36}$. The third number is the other solution of the equation.

After obtaining the solution, the author of the manuscript includes an interesting diagram to demonstrate step 3, which we will analyze in detail below (see the second example in Section 5). He also uses diagrams for checking that the solution is correct and that $15+\sqrt{189}$ plus $15-\sqrt{189}$ equals 30. All this is explained in a very detailed way and does not appear in the *Summa* or in the *Pràtica mercantívol*. The diagrams and the style of the explanations that go with them strongly suggest a teaching purpose of the manuscript.

Let us now turn to Ventallol's resolution. First he indicates that [by property $(*)$] the second quantity is $\sqrt{36}$, which is 6, and he adds that if the given number is not a perfect square, the problem has to be solved by *art maior* (algebra), a subject which would be "very difficult to give or to understand." Then he calculates $\frac{30}{2}\pm\sqrt{(\frac{30}{2})^2-36}$ to obtain the other two quantities. Finally, he states that checking the solution has to be done by *art maior*.[66]

It is remarkable that the solution given in the manuscript and that of Ventallol are the same, while Pacioli gives a different one, also correct: $105-\sqrt{10{,}989}$, 6, $105+\sqrt{10{,}989}$. This is important evidence for the common authorship of the manuscript and the *Pràtica mercantívol*. Pacioli does not explain anything about dividing 30 in two parts. He just declares that one has to operate by algebra, with the help of the second Book of Euclid. Later in the *Summa* (ff. 91r–v), the problem is solved (also for $q=36$) with the restriction $a+b+c=a\cdot b\cdot c$ $(**)$. Again, $b=6$. Pacioli solves the problem of finding a and c by two methods: one with two unknowns[67] and the other taking $x=a+c$ as the unknown and then obtaining $a=\frac{x}{2}-\sqrt{(\frac{x}{2})^2-36}$ and $c=\frac{x}{2}+\sqrt{(\frac{x}{2})^2-36}$. When these are substituted in $(**)$ and the resulting equation is solved, one finds $x=210$ and thus $a=105-\sqrt{10{,}989}$ and $c=105+\sqrt{10{,}989}$. It seems that Pacioli used one of these methods in the first, more general case. In a time when few people were interested in finding *all* the solutions of a problem, a restriction in its statement would not matter if it led to a result that was valid for the general case.

---

65 This property is not demonstrated by Pacioli, nor in the manuscript or in the *Pràtica mercantívol*. Again using modern notation, if we write $a=k\cdot r^2$, $b=k\cdot r$, and $c=k$, substitute these expressions in the initial condition and operate on the resulting expression, it follows that $q=(k\cdot r)^2$.

66 As can also be seen in Pacioli's *Summa*, "to proceed by means of algebra" was considered to work not only with unknowns in a strict sense, but also with quantities that are, or contain, nonexact radicals.

67 This resolution is explained by Rankin [1992, 306–307].

Ventallol could have written the manuscript and then included the description of the method in his printed arithmetic, without explaining its algebraic origin. This was not unusual at the time. It seems that these methods had to be learned by heart and just applied to very similar problems if the student did not have any knowledge of algebra.

5. Diagrams for equations

In several problems of Ms. 71 of Sant Cugat a very remarkable kind of diagram is used to construct the necessary equation and to reduce it to one of the six basic forms. These diagrams provide a short method to add or multiply polynomials and certainly show an advance toward systematization of the algebraic operations. The author uses them "to put the question in practice" (or "to solve the problem in practice").

The diagrams do not appear in Pacioli's *Summa* and, as far as we know, they have not been found in other previous or contemporary works. The following are some examples of the notations used in the manuscript, preceded by their modern equivalencies:

$$\begin{array}{cccccccc} x & 1 & \quad x^2 & 1 & \quad x^3 & 1 & \quad 2x^2+5x+3 & 2\\ & 0 & & 0 & & 0 & & 5\\ & & & 0 & & 0 & & 3\\ & & & & & 0 & & \end{array}$$

The notation is also used with fractional and negative coefficients. It must be noted that there is no formal introduction or explanation of these notations. They just appear in a number of problems leading to first-, second-, or third-degree equations. Now we will see how they are used in two examples.

In the first example, a partnership of three persons has won 100 pounds. The first invested a certain amount of money, the second invested twice as much plus two [pounds], and the third invested as much as the "surface" [i.e., product] of the amounts invested by the other two. The first partner must have 10 pounds of the gain.[68] The question is how much each person has invested.[69]

The author first follows Pacioli to explain the resolution of the problem: "Put that the first invested one thing (*cosa*), the second 2 things plus 2, the third 2 *census* plus 2 things. Now add all together, it makes 5 things plus 2 *census* plus 2 for the heart [sic; it stands for the total investment] of the partnership. Afterward you will say: if 5 things plus 2 *census* plus 2 win 100 pounds, how much will a thing win? It will win 100 things over 5 things plus 2 *census* plus 2. This will be equal to 10, which you know that belongs to him [the first partner]. Follow the equation. The thing will be $1\frac{1}{4}$ plus $\sqrt{\frac{9}{16}}$, that is 2; and so much the first invested. The second [invested] 6 and the third [invested] 12, etc."[70]

[68] It is implicitly assumed that the gains of the partners are proportional to their investments.

[69] "Tres fan compenya e an gonyat 100 lliures. Lo primer mes una quantitat; lo segon mes dos tants més 2; lo terç mes la superfícia del primer y del segon. Lo primer deu haver del guany 10 lliures. Jo us deman quant mate quascú" [Ms. 71, f. 100r]. The problem also appears in [Pacioli, 1494, f. 151v].

[70] "Posa que lo primer mes .1.co., lo segon 2.co. més 2, al terç mate 2.ce. més 2.co. Are sume-u tot ensemps, fa 5.co. més 2.ce. més 2 per lo cor de la compenya. Aprés diràs: Sy 5.co. més 2.ce. més .2. guanyen 100 lliures, què gonyara .1.co.? Gonyaran 100 co. esimi de 5 co més 2.ce. més 2. Asò serà egual a 10, que saps que li toca. Sagueix la aqueció. La .co. valrà 1¼ més ℞ 9/16, ço és 2; e tants mes lo primer. Lo segon 6 y lo terç 12, etc." [Ms. 71, f. 100r].

Then he includes the following diagram:

											egual			[egual]
		2	2					2		20			1	
1	2	2	5		1	100		5		50	100	50	0	2½
0	2	0	2	100	0	0		2	10	20	0	0	1	0

Each rectangle of the diagram stands for a polynomial. A translation into modern algebraic language of the rectangles from left to right yields the following series of operations[71]:

1. x (the capital of the first partner).
2. $2x + 2$ (the capital of the second partner).
3. $2x^2 + 2x$ (the capital of the third partner).
4. Addition of 1, 2 and 3 ($= 2x^2 + 5x + 2$) (the total capital of the partnership). As we see, the addition of polynomials is straightforward because of the positional value of the coefficients.

Now, if a capital of x receives a benefit of 10, a capital of $2x^2 + 5x + 2$ must receive 100. So the rule of three is applied and produces the equation $20x^2 + 50x + 20 = 100x$ (steps 5 to 11):

5. 100.
6. x.
7. Multiplication of 100 and x $(= 100x)$.
8. $2x^2 + 5x + 2$.
9. 10.
10. $\boxed{(2x^2 + 5x + 2) \cdot 10 =}$ $20x^2 + 50x + 20$.
11. $= 100x$.
12. $\boxed{20x^2 + 20 =}$ $50x$ ($50x$ is subtracted from both sides).
13. $x^2 + 1$.
14. $\boxed{=}$ $2\,½x$.

Notice that the equation is eventually normalized by dividing it by 20: $20x^2 + 20 = 50x \rightarrow x^2 + 1 = 2½x$. Now the equation can be solved by the "sixth rule" explained on ff. 58r–v, since it belongs to the sixth canonical type.

Notice that the diagram shows an empty rectangle between the seventh and eighth steps. This probably has something to do with an explanation that follows the diagram and refers to its second half: "The practice is as you see illustrated above. Following the letter, when you will be there where it says "he will win 100 things over 5 plus 2 *census* plus 2 and that will be equal to 10," you will proceed in this way in order to follow the equation: multiply 10 by 5 things plus 2 *census* plus 2 and you will find 50 things plus 20 *census* plus 20. And now follow the chapter: first subtract 50 things from 100 things, it leaves 50 things. Reduce it all to 1 *census*, divide 20 plus 20 *census* by 20, it will become 1 plus 1 *census*.

[71] Expressions in boxes will be added for explicative purposes.

Divide the 50 things by 20 as well, it will become 2 things and a half, and so you have that the first part is 1 *census* plus one and the second [part] $2\frac{1}{2}$ things.”[72] The author goes on to explain how to solve this final equation.

In our second example we will see how a multiplication with polynomials is carried out. The diagram is used to find an equation needed for dividing 30 in two parts such that their product is 36[73]; it is used in the problem on proportions discussed above. The following short comment is given before the diagram is set forth (see the transcription in Appendix A): “You will say: make 2 parts out of 30 such that multiplied one by the other it becomes 36. Do it by algebra: put that one part is 1 thing, the other 30 minus 1 thing. Multiply and equalize the extremes and you will find 30 things equal to 1 *census* plus 36 (...). The practice is such

				1 manco		1
1	manco	1	30	0	30	0
0						
		30	0		0	36

”

As we see, the word for minus (*manco*) appears sometimes before and sometimes after the number. The parts of 30 are x and $30 - x$. First of all they must be multiplied and then their product must be made equal to 36. Again, we have made a modern translation of the consecutive polynomials from left to right:

1. x.
2. $30 - x$.
3. $\boxed{x \cdot 30 =}$ $30x$ (the first column is multiplied by 30).
4. $\boxed{x \cdot (-x)} = -x^2$ (the first column is multiplied by -1).

The column indicating the polynomial $30x - x^2$, which results from adding 3 and 4 and gives the result of the product of 1 by 2, does not appear explicitly in the diagram.

5. $30x$.
6. $\boxed{=}\ x^2 + 36$.

The final two columns of the diagram together represent the equation $30x = x^2 + 36$, which is of the sixth basic type and thus can be solved by a specific rule.

We have thus seen how the diagrams for equations in Ms. 71 of Sant Cugat give a quick overview of the algebraic operations that are necessary in the resolution of a problem. But first of all they provide a very efficient method to perform those operations.

[72] “La pràtica és com veus dalt afigurat. Seguint la letra, quant seràs alí hon diu gonyarà 100.co. esimi de 5 co. més 2 ce. més 2 e que serà egual a 10, per a seguir la equació faràs axí: moltiplica 10 per 5.co. més 2 ce. més 2 e trobaràs 50 co. més 20 ce [més 20]. Y are segueix lo capítol, leva primer 50 co. de 100.co., resta 50 [.co.] Redueix-ho tot a .1.ce., parteix 20 per 20.ce. [*It should be* 20 més 20.ce. per 20], vindra-te’n .1. més 1 ce. Parteix les 50.co. axí mateix per 20, vindran-te’n 2 co e mitge e axí tens de la huna part 1.ce. més 1, de la altra 2½.co.” [Ms. 71, f. 100r].

[73] Ms. 71, f. 17v.

6. Conclusion

The Catalan manuscript Ms. 71 of Sant Cugat (*Arxiu de la Corona d'Aragó*, Barcelona) contains a very interesting set of notes on algebra and arithmetic, mainly based on Pacioli's *Summa de Arithmetica, Geometria, Proportioni et Proportionalità* (Venice, 1494). The manuscript pre-dates by about 20 or 30 years Marco Aurel's *Libro Primero de Arithmética Algebrática* (Valencia, 1552), which is the earliest known treatise on algebra printed in the Iberian peninsula. Not taking into account the original manuscript (apparently from the 1530s) of Pedro Nunes' algebra treatise printed in 1567,[74] Ms. 71 of Sant Cugat is, as far as we know, the only algebraic manuscript in a vernacular Iberian language that pre-dates Aurel's book. It gives us an idea of how algebra and commercial arithmetic were taught before the publication of the first independent algebra treatises, such as those by Cardano or Nunes. The author of the manuscript turns to Pacioli's *Summa* for topics such as irrational quantities or the theory of proportions and, above all, for equations and problems that are solved algebraically. On the other hand, he obtains the material on commercial arithmetic from French, Provençal, Catalan, Castilian, or Italian sources. The style of the text and the presence of an important section on commercial arithmetic point to an author who comes from the vernacular tradition of commercial arithmetic, although he is also interested in topics which lie outside of that tradition and shows at least some knowledge of Latin. It is clear that he understood very well what he was studying, and his algebraic skills are best exemplified by his use of diagrams of coefficients to work with equations, which are not found in any other known contemporary or earlier sources and were undoubtedly advanced for his time. We have summarized various pieces of evidence that the author of the notes in the Sant Cugat manuscript is the Majorcan Joan Ventallol, who wrote a commercial arithmetic titled *Pràtica mercantívol* (Lyon, 1521). If he is in fact the author, his position in the history of mathematics should be revised to consider him also a skillful algebraist.

Acknowledgments

First of all I thank Antoni Malet, who initiated the research project on Catalan manuscripts on commercial arithmetic within which I carried out the work for my Ph.D. dissertation. He encouraged me to make this contribution to the International Workshop "The Origins of Algebra: From al-Khwarizmi to Descartes" in Barcelona (March 2003), made comments on a preliminary version of this article, and suggested various corrections. I gratefully acknowledge the assistance of Jaume Riera, whose help was crucial in dating the manuscript studied here and the other Catalan manuscripts on arithmetic that I have mentioned. Many thanks are also due to the staffs of the Arxiu de la Corona d'Aragó, the Biblioteca de la Universitat de Barcelona, and the Biblioteca Nacional in Madrid for their assistance in relation to Ms. 71 of Sant Cugat, Andrés' and Aurel's treatises, and Ortega's book, respectively. Joan Antoni Mesquida provided me with a copy of his Ph.D. dissertation on the linguistic aspects of Ventallol's *Pràtica mercantívol*. Albert Presas supplied me with a copy of Pacioli's *Summa*, and Marie-Hélène Labarthe sent me a copy of *Le Kadran aux marchans*. Rosa Maria Aroso gave me some useful information concerning

[74] The *Libro de Álgebra en Arithmética y Geometría* (Antwerp, 1567) was printed in Spanish but Nunes claims that he had originally written it in Portuguese 30 years earlier, which brings us back to the 1530s (see [Leitão, 2002; Martyn, 1996]).

Portuguese arithmetics. I am grateful to the anonymous referees and to Benno van Dalen for their useful comments, suggestions, and corrections.

Appendix A

The following are transcriptions of the resolution of a problem concerning proportionality in the versions of the Catalan manuscript, Joan Ventallol's *Pràtica mercantívol*, and Pacioli's *Summa*.

Ms. 71 de Sant Cugat (Arxiu de la Corona d'Aragó):

> De tres quantitats contínues proporcionals havent a partir alguna altra quantitat per cascuna d'elles, e que los advaniments ajan de fer ho de ésser eguals a la suma del conjunt de dites tres quantitats contínues proporcionals, lavo[r]s forçadament de necessitat una d'elles a de ésser la ℞ de aquella tal quantitat que vols partir, ço és la segona. Perquè segueix lo corralari que partint dita quantitat per la primera lo que vindrà a de ésser egual a la terça et e converso, que pertint per la 3ª lo adveniment serà egual a la primera. E açò se verifica en totes. Sien les 3 quantitats proporcionals contínues quals se vullen e que per cascuna d'elles se aje de partir 36, que los adveniments ensemps eian de fer lo conjunt de dites quantitats. Dic que forçadament la segona ha de ésser (...) la ℞ de 36, que és la quantitat que volem partir. E puys as trobada la segona pots trobar la primera e terça aponente per força de les retgles desús dades. Emperò que deus trobar dos extrems, // la un manor de 6 e l'altra maior de 6, que ab 6 se agen en contínua proporcionalitat. E diràs fes de 30 dues parts que moltiplicada una per altra vinga de dita moltiplicacio 36. Fes per àlgebra. Posa que la una part sia 1.co., l'altra 30 manco .1.co. Moltiplica y eguala los extrems e trobaràs .30.co eguals a .1.ce. més 36. Segueix lo capítol. Hauràs la valor de la .co. 15 manco ℞ 189, e tant deus posar per lo primer nombre. Lo segon 6. Lo terç 15 més ℞ 189. E axí as fet de 36 tres parts en contínua proporció, que partit 36 per cascuna e los adveniments junts ensemps faran 36. La pràtica és tall. (...)[75]

Ventallol's *Pràtica mercantívol*:

> De tres quantitats contínues proportionals avent a partir alguna altra quantitat per cascuna d'eles e que los adveniments aien de ésser eguals a la suma del conjunt de dites tres quantitats propor[cionals], lavores de necessitat la una a de ésser la rays de aquela tal quantitat perquè segueix-se lo corelari que pertint dita quantitat per lo primer, lo que vindrà a de ésser egual a la terce et e converso. E si en les .3. quantitats contínues proportio[nals] qualssevulla e que per cascuna d'eles se aie a pertir .36. que los adveniments ensemps aien de fer lo coniunt de dites quantita[t]s, dic que forçadament la segona a de ésser la rays de .36., que és .6. Emperò si lo nombre que s'a de partir no té rays perfeta, la tal retgle seria de l'art maior, que se[r]ia molt difícil de donar ho entendra.
>
> E puys as trobada la segona quantitat, que és .6., pots trobar les altres per les retgles desús dites. Emperò deus trobar dos extrems: lo un maior de .6. e l'altra menor de .6., que ab .6. se aien en contínua proporció. E per trobar-los faràs així: trau .6. de .36., resten .30. Are digues: fes-me de .36. dues parts que moltiplicada una per altra facen .36. Mitade los 30, serà .15. Moltiplica'ls per si, fan .225. Trau-ne lo nombre que vols que face, ço és .36., resta .189. E la rays de .189., aiustant-hi la mitat de .30., que és .15., serà lo maior nombre. Lo menor serà la rays de .189. treta de .15. E així respondràs que lo primer nombre és .15. manco rays 189., e lo segon és .6. E lo terç és 15 més rays .189. E si per aquestes 3 quantitats perteix .36., ne

[75] Ms. 71, f. 17r–v. The text is followed by some illustrative diagrams and explanations. One of these diagrams has been discussed above.

> vindran aquelles mateixes parts que pertint per la maior ne vindrà la menor. E pertint per la menor ne vindrà la maior. E si perteix[e]s per la segona ne vindrà ella mateixa. La prova emperò se té a fer per l'art maior.[76]

Pacioli's *Summa*:

> E .3. quantità continue proportionali havendo a partire alcuna quantità altra per ciascuna desse, e che li avenimenti habino a essere equali a la summa over congionto de ditte .3. quantità continue proportionali, allora de necessità una de le ditte 3. quantità continue proportionali sera la radici de quella tal quantità che tu per quelle intendi partire: cioè sempre la .2ª. sia. ℞. de quella. Unde sequita poi el correlario che partendo ditta quantità per la prima proportionale l'avenimento converrà essere equale a la .3ª. proportionale et econverso, partendola per la .3ª. proportionale l'avenimento converrà essere equale alla prima, aciò gionte con la .2ª. facia la ditta summa e questo sia vero in tutte. Exemplum: sienno le .3. quantità proportionali la prima, seconda, terça qual si voglino e sienno continue. E che per ciascuna di loro se habi a partire .36. e che li avenimenti gionti asiemi habino a fare el congionto de ditte .3. quantità proportinali. Dico che la .2ª. di quelle converrà che sia .6., cioè la .℞ 36., che è la quantità che tu voli così partire. E mai falla. E poi che tu haverai trovato la .2ª. porrai per te stesso trovare la prima e la terça aponendote per força delle regole sopra date de le .3. quantità continue propertionali. Pero che tu harai a trovare doi extremi: uno menore de .6. e l'altro magiore, che con .6. se habino in continua proportionalità, quelle se rechiede per li havenimenti. E dirai trovame doi quantità che multiplicata una in l'altra facia .36., cioè quanto la .2ª. in sé. E partito .36. per ciascuna li havenimenti gionti facino la summa de ditte .3. quantità. Operando per algebra con l'aiuto del .2º. d'Euclide (como te dirò in le quantità semplici) haverai l'uno de li extremi essere .105. $\tilde{m}$ ℞. 10989. E sirà la prima quantità menore de .6. E l'altro sirà .105. $\tilde{p}$. ℞ 10989. e sirà la .3ª. E partito .36. per ciascuna desse li havenimenti aponto faranno .216. E tanto ancora sia el congionto de le .3. quantità ditte. Ut operando invenies. Onde partendo .36. per la prima che è .105. $\tilde{m}$ ℞. 10989. ne ven. 105. $\tilde{p}$. ℞ 10989., che è la .3ª. E partendo .36. per la .2ª. ne ven .6. E partendo .36. per .105. $\tilde{p}$.℞ 10989., che è la .3ª., ne ven .105. $\tilde{m}$ ℞. 10989. che è la prima. Si che li havenimenti sonno simili ali divisori. Commo in la maraveglia de .3. quantità continue proportionali te dissi. Ideo tu ipse concludes.[77]

References

Manuscript sources

Anonymous. Ms. 454, Biblioteca de Catalunya, Barcelona. Ca. 1390.
Anonymous. Ms. 102 (A.III 27), Biblioteca degl'Intronati di Siena. Ca. 1445.
Anonymous. Ms. 71 de Sant Cugat, Arxiu de la Corona d'Aragó, Barcelona. Ca. 1520.
Certain, Jehan. *Le Kadran aux marchans*. Ms. 2904, Bibliothèque de l'Arsenal, Paris. 1485.
Chuquet, Nicolas. *Triparty en la science des nombres*. Ms. fds franç. 1346, Bibliotèque Nationale de Paris, ff. 2r–147r. Lyon, 1484. Edited in [Marre, 1880].
Chuquet, Nicolas. *Appendix of the Triparty en la science des nombres*. Ms. fds franç. 1346, Bibliotèque Nationale de Paris, ff. 148r–210r. Lyon, 1484. Partially edited in [Marre, 1881].
Montmany, Gaspar et al. *Llibre que esplica lo que ha de ser un bon mercader*. Diversos 37B/2, Arxiu del Regne de Mallorca, Palma de Mallorca. Ca. 1490.

76 [Ventallol, 1521, ff. 117v–118r].
77 [Pacioli, 1494, ff. 88v–89r].

Printed sources

Andrés, J., 1515. Sumario breve de la prática de la aritmética de todo el curso del arte mercantívol bien declarado el qual se llama maestro de cuento. Juan Jofre/Vicent García Editores, Valencia. Facsimile edition, 1999.

Arrighi, G., 1965. Il codice L.IV.21 della Biblioteca degl'Intronati di Siena e la "Bottega d'abaco" a Santa Trinita in Firenze. Physis 7, 369–400.

Arrighi, G., 1965–1967. Un "programma" di didattica di matematica della prima meta del Quattrocento. Atti e Memorie dell'Accademia Petrarca di Lettere. Arti e Scienze di Arezzo, Nova Serie 38, 112–128.

Arrighi, G., 1982. Due scritti matematici catalani (sec. XV). Dal Ms. 102 (A.3.27) della Biblioteca degli'Intronati di Siena. La Fardelliana I (2–3) Trapani (Sicilia), 13–27.

Aurel, M., 1552. Libro Primero de Arithmética Algebrática. Joan de Mey, Valencia.

Beaujoan, G., 1988. The place of Nicolas Chuquet in a typology of fifteenth-century French arithmetics. In: Hay, C. (Ed.), Mathematics from Manuscript to Print 1300–1600. Clarendon, Oxford, pp. 73–88.

Benoit, P., 1982. La formation mathématique des marchands français à la fin du Moyen Age, l'exemple du Kadran aux marchands. In: Les entrées de la vie: Initiations et apprentissages: XIIe congrès de la Société des historiens médiévistes de l'Enseignement supérieur public, Nancy, 1981. Presses Universitaires, Nancy, pp. 209–224.

Benoit, P., 1988. The commercial arithmetic of Nicolas Chuquet. In: Hay, C. (Ed.), Mathematics from Manuscript to Print 1300–1600. Clarendon, Oxford, pp. 96–116.

Benoit, P., 1989. Calcul, algèbre et marchandise. In: Serres, M. (Ed.), Eléments d'histoire des sciences. Bordas, Paris, pp. 196–211.

Caunedo, B., Córdoba, R., 2000. El arte del alguarismo : Un libro castellano de aritmética comercial y de ensayo de moneda del siglo XIV. Junta de Castilla y León, Consejería de Educación y Cultura, Valladolid.

Cifuentes, Ll., 2002. La ciència en català a l'Edat Mitjana i el Renaixement. Universitat de Barcelona/Universitat de les Illes Balears, Barcelona/Palma de Mallorca.

Davis, N.Z., 1960. Sixteenth-century French arithmetics on the business life. J. Hist. Ideas 21, 18–48.

De la Torre y Del Cerro, A., 1971. Documentos para la Historia de la Universidad de Barcelona. I. Preliminares (1289–1451). Introduction, notes and comments by Jorge Rubió Balaguer. Universidad de Barcelona, Facultad de Filosofía y Letras.

Docampo, J., 2004. La formación matemática del mercader catalán 1380–1521 Análisis de fuentes manuscritas. Ph.D. dissertation. Universidade de Santiago de Compostela.

Ferrer, T., 1980. Els italians a terres catalanes (segles XII–XV). Anuario de Estudios Medievales 10, 393–467.

Flegg, G., 1988. Nicolas Chuquet—an introduction. In: Hay, C. (Ed.), Mathematics from Manuscript to Print 1300–1600. Clarendon, Oxford, pp. 59–72.

Franci, R., Toti Rigatelli, L., 1982. Introduzione all'Aritmetica Mercantile del Medioevo e del Rinascimento. Quattro Venti, Urbino.

Franci, R., Toti Rigatelli, L., 1985. Towards a history of algebra from Leonardo of Pisa to Luca Pacioli. Janus 72, 17–82.

Franci, R., Toti Rigatelli, L., 1988. Fourteenth-century Italian algebra. In: Hay, C. (Ed.), Mathematics from Manuscript to Print 1300–1600. Clarendon, Oxford, pp. 11–29.

Goldthwaite, R.A., 1972. Schools and teachers of commercial arithmetic in Renaissance Florence. J. Eur. Econ. Hist. 1, 418–433.

Hernando i Delgado, J., 2002. Del llibre manuscrit al llibre imprès. La confecció del llibre a Barcelona durant el segle XV. Arxiu de textos catalans antics 21. Documentació notarial, Barcelona, pp. 257–603.

Itard, J.s.v., 1979–1980. Chuquet, Nicolas. In: Gillispie, C.C. (Ed.), Dictionary of Scientific Biography, vol. 3. Scribner's, New York, pp. 272–278.

Labarthe, M.-H., 2004. Premières arithmétiques imprimées des Espagnes: Une hiérarchie des problèmes au service des procédés de résolution. Thèse doctorale. Université Paul Sabatier, Toulouse III.

Lafont, R. (Ed.), Tournerie, G., 1967. Francés Pellos. Compendion de l'abaco. Transcription and philological comment by Robert Lafont and mathematical comment by Guy Tournerie. Edition de la Revue des Langues romanes, Montpellier.

Leitão, H., 2002. Sobre as "Notas de Álgebra" atribuídas a Pedro Nunes (ms. Évora, BP, Cod. CXIII/1-10). Euphrosyne 30, 407–416.

López Piñero, J.M[a]., 1979. Ciencia y Técnica en la Sociedad española de los siglos XVI y XVII. Labor, Barcelona.

López Piñero, J.M[a]., Bujosa, F., 1984. Los impresos científicos españoles de los siglos XV y XVI. Inventario, bibliometría y thesaurus, vols. 2 and 3. (Inventario D–Q). Universidad de Valencia, Valencia.
Malet, A. (Ed.), 1998. Francesc Santcliment. Summa de l'art d'aritmètica. Introducció, transcripció i notes a cura d'Antoni Malet. Eumo editorial, Vic.
Malet, A., Paradís, J., 1984. Els orígens i l'ensenyament de l'àlgebra simbòlica. Edicions de la Universitat de Barcelona, Barcelona.
Martyn, J.R.C. (Ed., trans.), 1996. Pedro Nunes (1502–1578). His Lost Algebra and Other Discoveries. Peter Lang Publishing, New York.
Marre, A. (Ed.), 1880. Notice sur Nicolas Chuquet et son *Triparty en la science des nombres*. Bulletino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche Publicato da B. Boncompagni 13, 555–659, 693–814.
Marre, A. (Ed.), 1881. Appendice au *Triparty en la science des nombres* de Nicolas Chuquet parisien. Bulletino di Bibliografia e di Storia delle Scienze Matematiche e Fisiche Publicato da B. Boncompagni 14, 413–460.
Mesquida, J.A., 1996. La *Pràtica mercantívol* de Joan Ventallol. Estudi lingüístic i edició d'un text matemàtic del segle XVI. Ph.D. dissertation. Universitat de les Illes Balears.
Miquel, F.X., 1937. Catàleg dels llibres manuscrits de la Biblioteca del Monestir de Sant Cugat del Vallès existents a l'Arxiu de la Corona d'Aragó. Imprenta de la Casa de Caritat, Barcelona.
Navarro Brotóns, V., Salavert Fabiani, V.L., Rosselló Botey, V., Darás Román, V., 1999. Bibliographia physico-mathematica hispanica (1475–1900). Volumen I. Libras y folletos 1475–1600. Universidad de Valencia-CSIC, Valencia.
Nunes, P., 1567. Libro de Álgebra en Arithmética y Geometría. Heirs of Arnold Birckman, Antwerp.
Ortega, J. de, 1512. Síguese una compusición de la arte de la arismetica y juntamente de geometría. Joan Trinxer, Lyon.
Pacioli, L., 1494. Summa de Arithmetica, Geometria, Proportioni et Proportionalità. Paganino de Paganini, Venezia.
Pellos, F., 1492. Compendion de l'abaco. N. de Benedetti and J. Suigo, Turin. Modern edition in [Lafont and Tournerie, 1967].
Pla, J., 1998. Panoràmica del sistema decimal posicional des dels origens indis a l'arismètica de Santcliment. Calligraphia et tipographia, arithmetica et numerica, chronologia 7, 101–256.
Rankin, F.K.C., 1992. The Arithmetic and Algebra of Luca Pacioli (c. 1445–1517). Ph.D. dissertation. The Warburg Institute, University of London.
Salavert, V., 1985. Influencias e informantes en la aritmética práctica valenciana del siglo XVI. In: Báguena, M.J., et al. (Eds.), Estudios sobre la medicina y la ciencia valencianas: Siglos XVI–XIX. Cátedra de Historia de la Medicina, Valencia, pp. 155–178.
Salavert, V., 1990. Introducción a la historia de la aritmética práctica en la Corona de Aragón en el siglo XVI. Dynamis 10, 63–91.
Santcliment, F., 1482. Summa de l'art d'Aritmètica. Pere Posa, Barcelona. Modern edition in [Malet, 1998].
Sesiano, J., 1984. Une arithmétique médievale en langue provençale. Centaurus 27, 26–75.
Sigler, L. (Trans.), 2002. Fibonacci's *Liber abaci*: A Translation into Modern English of Leonardo Pisano's Book of Calculation. Springer-Verlag, New York.
Spiesser, M., 2000. Problèmes linéaires dans le *Compendy de la praticque de nombres* de Barthélemy de Romans et Mathieu Préhoude (1471): Une approche nouvelle basée sur des sources proches du *Liber abbaci* de Léonard de Pise. Historia Math. 27, 362–383.
Spiesser, M., 2003. Les manuels d'arithmétique pour les marchands dans la France du XVe siècle. Bulletin de l'APMEP (=Association des Professeurs de Mathématiques de l'Enseignement Public) 444, 32–50.
Swetz, F.J., 1987. Capitalism and Arithmetic: The New Math of the 15th Century. Open Court, La Salle, IL.
Ulivi, E., 2001. Mariano del M[o] Michele, un maestro d'abaco del XV secolo. Nuncius: Annali di Storia della Scienza 16, 301–346.
Van Egmond, W., 1978. The earliest vernacular treatment of algebra: The "Libro di ragioni" of Paolo Gerardi (1328). Physis 20, 155–189.
Van Egmond, W., 1980. Practical Mathematics in the Italian Renaissance: A Catalog of Italian Abbacus Manuscripts and Printed Books to 1600. Instituto e Museo di Storia della Scienza, Florence.
Van Egmond, W., 1988. How algebra came to France. In: Hay, C. (Ed.), Mathematics from Manuscript to Print 1300–1600. Clarendon, Oxford, pp. 127–144.
Ventallol, J., 1521. Pràtica mercantívol. Joan de la Place, Lyon Palma de Mallorca. Facsimile edition, Talleres de la Antigua Imprenta Soler, 1985.